Assignment

02

Subject: Computer Codes & Arithmetic + Approximations and errors in   
 Computing .

Course Title : Numerical Methods & computation   
 Course No : CSE-205



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Computer Codes & Arithmetic  
 Errors in Arithmetic

In integer arithmetic , while all arithmetic operation are exact ,we  
might come across the following two situation :  
  
1. An operation may result in a large number that the computer can  
handle .  
2. An integer division may result in truncation of the remainder .  
 When the result is larger than the maximum limit ,its referred to  
as an overflow and when its less than the lower limit, its referred to as underflow . Unfortunately ,most computers do not issue any warnings  
or messages or integer overflow or underflow . Therefore , we should   
use integer arithmetic with utmost care .  
  
The floating point arithmetic system is prone to the following errors :  
  
 1. Error due to inexact representation of a decimal number in a binary form .For example , consider the decimal number 0.1 .The binary equivalent number of this number is 0.0001100110011…..The binary equivalent has a repeating fraction and therefore must be terminated at some point .  
  
2. Error due to rounding method used by the computer , in order to limit the  
number of significant digits . If the numbers added are too different in magnitude , the smaller may be treated as if it was zero .  
  
3. Floating point subtraction may induce a special phenomenon .It is possible that some mantissa positions in the result are unspecified .  
This happens when two nearly equal numbers are subtracted .this is  
 known as subtractive cancellation .If the operands themselves represent approximate values , the loss of significance is serious since it greatly reduces the number of significant digits .  
  
 4. Overflow or underflow can occure in floating point operations when   
the result is outside the limits of floating point number system of the computer .  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
   
 Approximation & Errors in Computing  
  
\*\*Introduction :  
 Approximations and errors are an integral part of human life .They  
are everywhere and unavoidable . This is more so in the life of a computational scientist .  
 We can notuse numerical methods and ignore the existence of errors . Errors come in a variety of forms and sizes ; some are avoidable , some are not . For example , data conversion and roundoff errors can not be avoidable , but a human error can be eliminated .

Total error|->Modeling error +Inherent error + Numerical error +Blunders   
error .  
  
\*\*Modeling Errors:  
 Mathematical models are the basis for numerical solutions .The are formulated to represent physical processes using certain  
parameters involved in the situation . In many situations , its impractical or impossible to include all of the real problem and therefore certain simplifying assumptions are made . For example , while developing a model for calculating the force acting on a falling body , we may not be able to estimate the air resistance coefficient properly or determine the direction and magnitude of wind force acting on the body , and so on .To simplify the model , we may assume that there is no wind force acting on the body . All such simplification certainly result in errors in the output from such models .  
  
  
\*\* Inherent Errors :  
 Inherent errors are those that are present in the data supplied to the model . Inherent errors (also known as input errors) contain two components , namely , data errors and conversion errors .  
  
\*Data Errors :  
 Data error (also known as empirical error) arises when data for a problem are obtained by some experimental means and are , therefore may be unavoidable . A physical measurement , such as distance , a voltage , or a time period , can’t be exact .  
  
 \*Conversion Errors:

Conversion errors (also known as representation errors ) arise due to the limitation of the computer to store the data exactly . We know that the floating point representation retains only a specified number of digits .The digits that are not retained constitute the roundofferror .

\*\*Numerical Errors :  
 Numerical errors(also known as procedural errors) are introduced during the process of implementation of a numerical method . They come in two forms ,roundoff errors and truncation errors . The total numerical error is the summation of these two errors .  
 \*RoundoffErrors :  
Rouundoff errors occurs when a fixed number of digits are used to represent exact number . Since the numbers are stored at every stage of computation ,roundoff error is introduced at the end of every arithmetic operation .Consequently , eventhough an individual roundoff error could be very small , the cumulative effect of a series of computations can be very significant .  
 \*Truncation Errors :  
In numerical analysis and scientific computing, truncation error is the error made by truncating an infinite sum and approximating it by a finite sum. For instance, if we approximate the sine function by the first two non-zero term of its Taylor series, as in \sin(x) \approx x - \tfrac16 x^3 for small x, the resulting error is a truncation error. It is present even with infinite-precision arithmetic, because it is caused by truncation of the infinite Taylor series to form the algorithm.  
Often, truncation error also includes discretization error, which is the error that arises from taking a finite number of steps in a computation to approximate an infinite process. For example, in numerical methods for ordinary differential equations, the continuously varying function that is the solution of the differential equation is approximated by a process that progresses step by step, and the error that this entails is a discretization or truncation error. See Truncation error (numerical integration) for more on this.

Ocasionally, round-off error (the consequence of using finite precision floating point numbers on computers) is also called truncation error, especially if the number is rounded by truncation.  
  
\*\*Blunders Erros :  
Blunders are errors that are caused due to human imperfection . As the name indicates , such errors may cause a very serious disaster in the result . Since these errors are due to human mistakes , it should be possible to avoid them to a large extent by acquiring a sound knowledge of all aspects of the problem as well as the numerical process .

Human errors can occur at any stage of the numerical processing cycle . Some common type of errors are -  
1. Lack of understanding of the problem .  
2. Wrong assumption .  
3. Overlooking of some basic assumptions required for formulating the model .  
4. Wrong guessing of initial values .  
5. Selecting a wrong numerical method for solving the mathematical method .  
  
\*\* Absolute and relative Errors :  
Absolute and relative error are two types of error with which every experimental scientist should

be familiar. The differences are important.

\*Absolute error:  
Absolute error is the amount of physical error in a measurement, period. Let’s say a meter stick is used to measure a given distance. The error is rather hastily made, but it is good to ±1mm. This is the absolute error of the measurement. That is,

absolute error = ±1mm (0.001m).

In terms common to Error Propagation

absolute error = Δx

where x is any variable.

\*Relative Error:  
Relative error gives an indication of how good a measurement is relative to thesize of the thing being measured. Let’s say that two students measure two objects with a meterstick. One student measures the height of a room and gets a value of 3.215 meters±1mm

(0.001m). Another student measures the height of a small cylinder and measures 0.075 meters ±1mm (0.001m). Clearly, the overall accuracy of the ceiling height is much better than that of the 7.5 cm cylinder. The comparative accuracy of these measurements can be determined by looking at their relative errors.